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SEARCH, OBSERVATION, AND ATTACK PROBLEMS

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A significant amount of research is being conducted on the cooperative behavior of multiple uninhabited combat aerial vehicles (UCAVs) in the area of search, observation, target recognition, and attack. The cooperative behavior must be carried out in a communications-limited, noisy, adversarial, and uncertain environment. It is envisioned that effective solutions of these problems will involve a combination of top-level operations research/artificial intelligence type decision making combined with distributed control, distributed estimation, and real-time trajectory optimization. The solutions of several problems which have importance in the cooperative behavior of multiple vehicles are presented.

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PREFACE

The work presented in this technical report was a joint effort between researchers at the Air Force Research Laboratory (AFRL) and USA National Research Council. The problems of search and attack of enemy targets are research and development.

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		b_1	b_2	b_3	

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Some Specific Search, Observation, and Attack Problems

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Abstract

A significant amount of research is being conducted on the cooperative behavior of multiple uninhabited combat aerial vehicles (UCAVs) in the areas of search, observation, target recognition, and attack. The cooperative behavior must be carried out in a communications-limited, noisy, adversarial, and uncertain environment. It is envisioned that effective solutions of these problems will involve a combination of top-level operations research/artificial intelligence type of decision making combined with distributed control, distributed estimation, and real-time trajectory optimization. The solutions of several problems which have importance in the cooperative behavior of multiple vehicles are presented.

Introduction

There are numerous books and articles on the topics of searching, observing, and detecting targets [1]-[15]. In [1] R. Murphey presents an approximate algorithm for a weapon target assignment stochastic program. In [2] he provides an introduction to collective and cooperative systems. In [3] he presents concept of operations for persistent area dominance. In [4] J. Cloutier and A. Bolonkin develop the search and attack strategies. In [5] Washburn presents a review of various search problems including localization, tracking, false alarms, false targets, optimal search, optimal planning, optimization of the detection threshold, etc. He also provides known techniques for their solution. In [6] editors Haley and Stone provide a collection of interesting articles about the above topics, which include applications in rescue, surveillance, exploration, medicine, industry, and clearance. In [7] Stone reviews methods of optimal planning, and searching for real and false targets. In [8] Brown studied optimal search for a moved target in discrete time and space. Steward [9] studied a constrained searcher motion. Kisi [10] researched optimal stopping of a search.

In [11] Enns et al addressed the problem of cooperative search in a horizontal plane subject to atmospheric and other disturbances and minimized the total path length traveled by the group of vehicles. In [12] Chandler et al investigated the complexity in cooperative control, implementing a hierarchical decomposition where team vehicles are allocated to subteams using set partition theory. Finally, in [13] Earl and D'Andrea used an optimization approach for synthesizing control strategies for cooperative multi-agent behavior.

In this paper we investigate some specific search, observation, and attack problems.

The US Air Force is planning to use uninhabited combat aerial vehicles (UCAVs) extensively for searching, and attacking enemy targets. These concepts involve the cooperative behavior of

multiple UCAVs operating in a noisy environment. The effective solutions of these problems involve a combination of top-level operations research/artificial intelligence type of decision making combined with distributed control, distributed estimation, and real-time trajectory optimization. It is doubtful that each vehicle will have global information. Rather, each vehicle will receive limited noisy information concerning the states of the other vehicles, and possibly only information concerning the states of its nearest neighbors.

Some of the specific problems that we will address are:

(1) Calculation of needed number of observation vehicles or observation period;

(2) Static Assignment problem of aircraft to targets;

(3) Minimum distance traveled for multiple vehicle observation and attack;

(4) Optimal observer location for the single attack vehicles;

(5) Quick annihilation of targets by attack vehicles located in different places.

The rest of the paper is organized as follows:

In section 1, the problem of needed number of observation vehicles is considered. The equations for computing the needed number of vehicles or observation period are presented. In section 2, the Static Assignment of aircraft to targets is presented along with methods of their solution. In section 3 Minimum Distance Traveled for Multiple Vehicles, equations and method of their solution are presented. In section 4, the problem of Optimal Observer Location for the Single Attack vehicle is studied. In section 5, the Problem of a quick annihilation of Targets by attack vehicles located in different places is presented.

The paper is then closed with a Note and Summary.

Problem 1. Calculation of Needed Number of Observation Vehicles or Observation Period.

Problem Statement: Consider an area to be observed S with observation period given by P. Assume that the vehicles are identical and have speed, V, and an observation field view with width l. The problem addressed in this section is, how many vehicles are needed for observing this area, such that any point in S is viewed at least every P units of time.

The number of required vehicles is

$$n = S/sP = S/lVP, \qquad (1.1)$$

where $s=lV$ is the area which one vehicle observes in unit time.

If there are M observation areas, the needed number of vehicles is

$$n = \sum S_j/sP = \sum S_j/lVP, \quad (j = 1,2,3,\ldots,M). \qquad (1.2)$$

If each search areas S_j has different period of observation P_j, then

$$n = \sum S_j/P_j/s = \sum S_j/\, P_j/lV, \quad (j = 1,2,3,\ldots,M). \qquad (1.3)$$

If there is a given number of vehicles n, we can find the period

$$P = S/\, lVn. \qquad (1.4)$$

If the vehicles have difference performance l, V, then

$$P = S/\sum l_i V_i \qquad (i=1,2,3,\ldots,n), \qquad (1.5)$$

or for some areas,

$$P = \sum S_j/\sum l_i V_i \quad, \quad (j=1,2,3,\ldots,M, \;\; i=1,2,3,\ldots,n). \qquad (1.6)$$

If every area has a different period of observation, then

$$\sum s_i = \sum l_i V_i = \sum S_j/P_j \quad, \quad (j=1,2,3,\ldots,M, \;\; i=1,2,3,\ldots,n), \qquad (1.7)$$

where P_j is period for area j.

Problem 2. Static Assignment Problem

Problem Statement:

Consider M vehicles and N independent target areas. If the vehicles are all identical, then we may define the decision variable x_j, as the number of vehicles assigned to area j, j=1,2,3,...,N.

Assume that a single vehicle engagement of a target area is independent of every other engagement. Then the outcomes of the engagement are independent Bernoulli distributed.

Let each target be assigned a positive real number V_j to indicate preference between areas. Assume that areas may be partitioned into classes and each class is unique to the decision maker. Our objective is to maximize the expected observation (searching, damage) to the areas which is equivalent to minimizing the expected area values.

The resulting integer programming formulation is:

$$\text{Minimize} \qquad \sum V_j q_j^{x_j} \quad j=1,2,3,\ldots,N. \tag{2.1}$$

Subject to

$$\sum x_j = M, \tag{2.2}$$

where x_j is an integer valued decision variable representing the numbers assigned to area j, and q_j are real numbers denoting the probabiliting of failure for a single engagement of area j. The equality constraint ensures that all of the vehicles are used in the assignment. Notice that nothing prevents all of the vehicles from being assigned to a single area.

The algorithm for this class of mathematical problems was suggested by den Broeder [2] (see also [1]). It includes the following steps:

Algorithm for solution:

Step 0. For each $j=1,\dots,N$, let $x_j=0$, and denote the probability of survival of target j by

$S_j=V_j q_j^{x_j}$. Initialize vehicle index $i = 1$.

While $i < M$ do

Step 1. Find area j for which aircraft i has greatest effect:

$$k = arg\ max\{S_j(1-q_j)\}.$$

Step 2. Add aircraft i to area k: $x_k=x_k+1$ and revise the probability of survival of target k: S_k

$= S_k q_k$.

$i = i + 1$.

See References [1],[14] for Problem 2.

Problem 3. Minimum Distance Traveled for Multiple Vehicle Observation and Attack

Problem Statement.

Suppose there is a group of observation vehicles and/or attack vehicles located on a base. There are n potential target locations and these sites need to be observed or attacked. The vehicles leave from a base indexed by 0, visit each of the n other places exactly once, and return to the base 0. The group uses t vehicles (here t may be allowed to vary). Every vehicle can visit no more than p places in one tour (by a tour we mean a succession of visits to places without stopping at base 0). It is required to find an itinerary that minimizes the total distance traveled by the group. For a given vehicle speed, this is equivalent to the minimum time to visit all locations, or perhaps minimum fuel consumption.

Note that if t is fixed, then for the problem to have a solution we must have $tp \geq n$. This is known as the t – traveling salesman problem, with constraints on cities per tour [17, p.431]. We have a GROUP of t vehicles (Fig.1).

Mathematical model.

Let d_{ij} ($i \neq j = 0,1,\ldots,n$) be the distance covered in traveling from place i to place j. We then have the following integer programming problem:

Minimize the linear form

$$\Sigma\Sigma d_{ij} x_{ij} \quad (0 \leq i \neq j \leq n) \tag{3.1}$$

over the set determined by the relations

$$\Sigma x_{ij} = 1 \quad j=1,\ldots,n \quad (\Sigma \text{ is } i=0,\ i \neq j,\ \ldots,\ n), \tag{3.2}$$

$$\Sigma x_{ij} = 1 \quad j=1,\ldots,n \quad (\Sigma \text{ is } i=0,\ i \neq j,\ \ldots,\ n), \tag{3.3}$$

$$u_i - u_j + p x_{ij} \leq p - 1, \qquad 1 \leq i \neq j \leq n, \tag{3.4}$$

where the x_{ij} are nonnegative integers and the u_i ($i = 1,\ldots,n$) are arbitrary real numbers which, as an approximation, can be restricted to nonnegative integers. If t is fixed, we must add the restriction

$$\Sigma x_{i0} = t \quad (i=1,\ldots,n). \tag{3.5}$$

Method of solution

From a theoretical point of view, the techniques of Gomory and others should enable us to solve any integer programming problem. The digital computer codes based on these algorithms have, in most instances, proved unpredictable in their ability to guarantee convergence to the optimum. The success of such algorithms varies according to the problem and the rules used to develop the cutting planes. This is in sharp contrast to the

highly successful use of the simplex method to solve almost any standard (continuous) linear problem. The reader is referred to Balinski [15] and Trauth and Woolsey [16] for discussions comparing the efficacy of different algorithms and related computational experience.

Problem 4. Optimal Observer Location for the Single Attack Vehicle

Statement of the problem.

Consider a region where there are n places in which a target must appear with probability p_i, and (or) importance a_i, $i=1,2,\ldots,n$. The coordinates of the sites are known. There is one attack vehicle which observes the entire region and has limited maximum speed. The problem: where is the optimal location of the vehicle to reach the probabilistic appearance of the target in minimal time (Fig.2).

Mathematical model and solution.

Let us denote the coordinates of the target sites as (x_i, y_i), $(i = 1,\ldots,n)$; the coordinates of the attack vehicle are x_o, y_o. The probability of the target appearance is p_i. The solution of this problem is to locate the vehicle at the mean of the 2-dimensional distribution at $\pi = \{p_1,\ldots,p_n\}$ and is given by equations:

$$x_o = (\Sigma p_i x_i)/\Sigma p_i\,, \quad i = 1,\ldots,n\,, \tag{4.1}$$

$$y_o = (\Sigma p_i y_i)/\Sigma p_i\,, \quad i=1,\ldots,n\,. \tag{4.2}$$

The target importance a_i may be a consideration and yields the optimal location:

$$x_o = (\Sigma a_i x_i)/\Sigma a_i\,, \quad i = 1,\ldots,n\,, \tag{4.3}$$

$$y_o = (\Sigma a_i y_i)/\Sigma a_i\,, \quad i=1,\ldots,n\,. \tag{4.4}$$

Similarly, both the probability and importance of the target can be considered together, which leads to the optimal location

$$x_o = (\Sigma p_i a_i x_i)/\Sigma p_i a_i, \quad i = 1,\dots,n, \tag{4.5}$$

$$y_o = (\Sigma p_i a_i y_i)/\Sigma p_i a_i, \quad i=1,\dots,n. \tag{4.6}$$

Note. The above solutions also represent the solutions for the optimal location for the base of an attack vehicle.

Problem 5. Quick Annihilation of Targets by Attack Vehicles Located in Different Places.

Statement of the problem.

Consider that there are some finite number of regions where a collection of attack vehicles may be located. These may be their bases or holding points in current time. Information is received that there are targets in some other regions located far from the current vehicle regions. Assume that the target locations are point locations, and that one vehicle can destroy one target.

The aim is to destroy all of the enemy targets in minimum group time (Fig.3).

The simpler example.

To illustrate the application of the mathematical description of the above problem, we first consider a simpler example. The general mathematical model and detailed discussion are given later.

A commander wishes to send a number of attack vehicles from several locations to a number of the target regions. Each enemy region requires a certain number of attack vehicles, while each base (location region of attack vehicles) can supply up to a certain number of vehicles. Let us define the following:

m = the number of bases (location regions of the attack vehicles),

n = the number of targets,

a_i = the total number of the attack vehicles available at base i,

b_i = the total requirement of the vehicles by enemy region j,

x_{ij} = the number of vehicles being sent from base i to enemy region j.

We shall assume that the total number available is equal to the total required, that is,

$$\Sigma a_i = \Sigma b_j \quad .$$

As will be shown later, this assumption is not a restrictive one.

The x_{ij} are the unknown vehicles to be determined. If we form the array (for m = 2 and n = 3)

Table #1

Enemy regions or targets

		1	2	3	
Base	1	x_{11}	x_{12}	x_{13}	a_1
Base	2	x_{21}	x_{22}	x_{23}	a_2
		b_1	b_2	b_3	

we see that the total number of vehicles being sent from base 1 can be expressed by the linear equation

$$x_{11} + x_{12} + x_{13} = a_1 \, . \qquad (5.1)$$

For base 2, we have

$$x_{21} + x_{22} + x_{23} = a_2 \, . \qquad (5.2)$$

We also note that the total number of vehicles being sent to the three enemy regions are expressed by the equations

$$x_{11} + x_{21} = b_1 \;,$$
$$x_{12} + x_{22} = b_2 \;, \qquad (5.3)$$
$$x_{13} + x_{23} = b_3 \;.$$

The commander knows the time (or distance) c_{ij} of flight of one vehicle from base i to target j. We have the additional assumption that the time relationship is linear; i.e., the cost of the mission x_{ij} is $c_{ij}x_{ij}$.

The commander wishes to determine how many vehicles should be sent from each base to each enemy region so that the total annihilation time (fuel consumption) is a minimum. The objective of minimizing the time is achieved by minimizing the linear time (common distance, common fuel consumption, drain, extension of resource) function

$$c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} \qquad (5.4)$$

Since a negative x_{ij} would represent a flight from target j to base i, we require that all of the variables $x_{ij} \geq 0$. We include the round trip in c_{ij} as a doubled value (to target and back to base).

By combining Eqs. (5.1) and (5.3), the objective function (5.4) and the condition of nonnegative of the variables, the problem for $m = 2$ and $n = 3$ can be formulated in terms of the following linear programming problem:

Minimize the cost function

$$c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} \qquad (5.5)$$

subject to the conditions $x_{ij} \geq 0$

and

$$x_{11} + x_{12} + x_{13} = a_1 \,,$$

$$x_{21} + x_{22} + x_{23} = a_2\,,$$

$$x_{11} + x_{21} = b_1\,, \tag{5.6}$$

$$x_{12} + x_{22} = b_2\,,$$

$$x_{13} + x_{23} = b_3\,.$$

General mathematical problem

The same attack vehicles are to be sent in the amounts $a_1, a_2, \ldots, a_m$, respectively, from each of m bases and are needed in the amounts $b_1, b_2, \ldots, b_n$, respectively by each of n attacked regions. The time (cost, etc.) of flight (to the target and back) of one vehicle from the i-th base to the j-th enemy region (target) is c_{ij} and is known for all combinations (i,j). The problem is to determine the number of vehicles x_{ij} to be sent over routes (i,j) so as to minimize the total cost (time) of the operation.

To develop the constraints of the problem, we set up Table 1. The amount vehicles from base i to the enemy region (target) j is x_{ij}; the total number of vehicles from base i is $a_i \geq 0$, and the total number of vehicles being sent to enemy region j is $b_j \geq 0$.

Here we temporarily impose the restriction that the total number of vehicles sent is equal to the total number of vehicles used, that is,

$$\Sigma a_i = \Sigma b_j = A\,. \tag{5.7}$$

The total cost of using x_{ij} vehicles over routes (i,j) is $c_{ij}\, x_{ij}$. Since a negative number of vehicles has no valid interpretation for problem as stated, we restrict each $x_{ij} \geq 0$. From the table, we have the mathematical statement of the problem: ***Find values for variables x_{ij} which minimize the total time (cost, group distance traveled, group fuel consumption, drain, extension of resource, etc.)***

$$\Sigma\Sigma c_{ij}x_{ij} \qquad i = 1,2,\ldots,m\ , \quad j = 1,2,\ldots,n \quad , \tag{5.8}$$

subject to the constraints

$$\Sigma x_{ij} = a_i \quad i = 1,2,\ldots,m \quad , \tag{5.9}$$

$$\Sigma x_{ij} = b_i \quad j = 1,2,\ldots,n \quad , \tag{5.10}$$

and

$$x_{ij} \geq 0\ . \tag{5.11}$$

Equations (2) represent the row sums of Table #1 and (3) the column sums. In order for Eqs. (2) And (3) to be consistent, we must have the sum of Eqs. (2) equal to the sum of Eqs. (3); that is,

$$\Sigma\Sigma x_{ij} = \Sigma\Sigma x_{ij} = \Sigma a_i = \Sigma b_j = A \qquad i = 1,2,\ldots,m \qquad j = 1,2,\ldots,n\ . \tag{5.12}$$

We note that the system of Eqs. (1) - (4) is a linear programming problem with $m + n$ equations in mn variables.

Method of Solution

The solution of this problem may be obtained using the standard simplex method or other special method, such as an interior point method.

Note

The difference between Problem 3 and Problem 5 is the type of attack vehicles being used:

In Problem 3 the attack vehicles can carry multiple missiles and one vehicle may be used for different targets during one flight.

In Problem 5, the attack vehicles are small and can carry only one missile for the destruction one target, but multiple vehicles may be used against one target. One vehicle can attack only one target and must then return to its base.

References:

1. R. A. Murphey, An Approximate Algorithm for a Weapon Target Assignment Stochastic Program. Approximation and Complexity in Numarical Optimization: Continuous and Discrete Problems (P.M. Pardalos, Editor),pp.1-2. 1999 Kluwer Academic Publishers.
2. R. Murphey, An Introduction to Collective and Cooperative Systems. Chapter 1. Air Force Research Laboratory, Munitions Directorate.
3. R. Murphey, Concept of Operations (CONOPS) for Persistent Area Dominance (PAD). Report AFRL/MNGN.
4. A. Bolonkin, J. Cloutier, Search of Enemy Targets. Report AFRL/MNGN, Eglin Air Force Base, Florida 32542-6810, USA. June 2002.
5. A.R.Washburn, Search and Detection, Informs.
6. Search Theory and Application, Edited by K.B.Haley and L.D.Stone, Plenum Press,1980, NY.
7. L. D. Stone, Theory of Optimal Search, ORSA,1975.
8. S.S. Broun, Optimal Search for a Moving Target in Discrete Time and Space with an Exponential Detection Function. Collection [6].
9. T.J. Steward, Experience With a Branch-and-Bound Algorithm for Constrained Searcher Motion. Collection [6].
10. T. Kisi, Optimal Stopping of the Investigation Search. Collection [6].
11. D. Enns, D. Bugajski, , and S. Pratt, Guidance and Control for Cooperative Search, Proceedings of the American Control Conference, Anchorage, AK, May 2002.

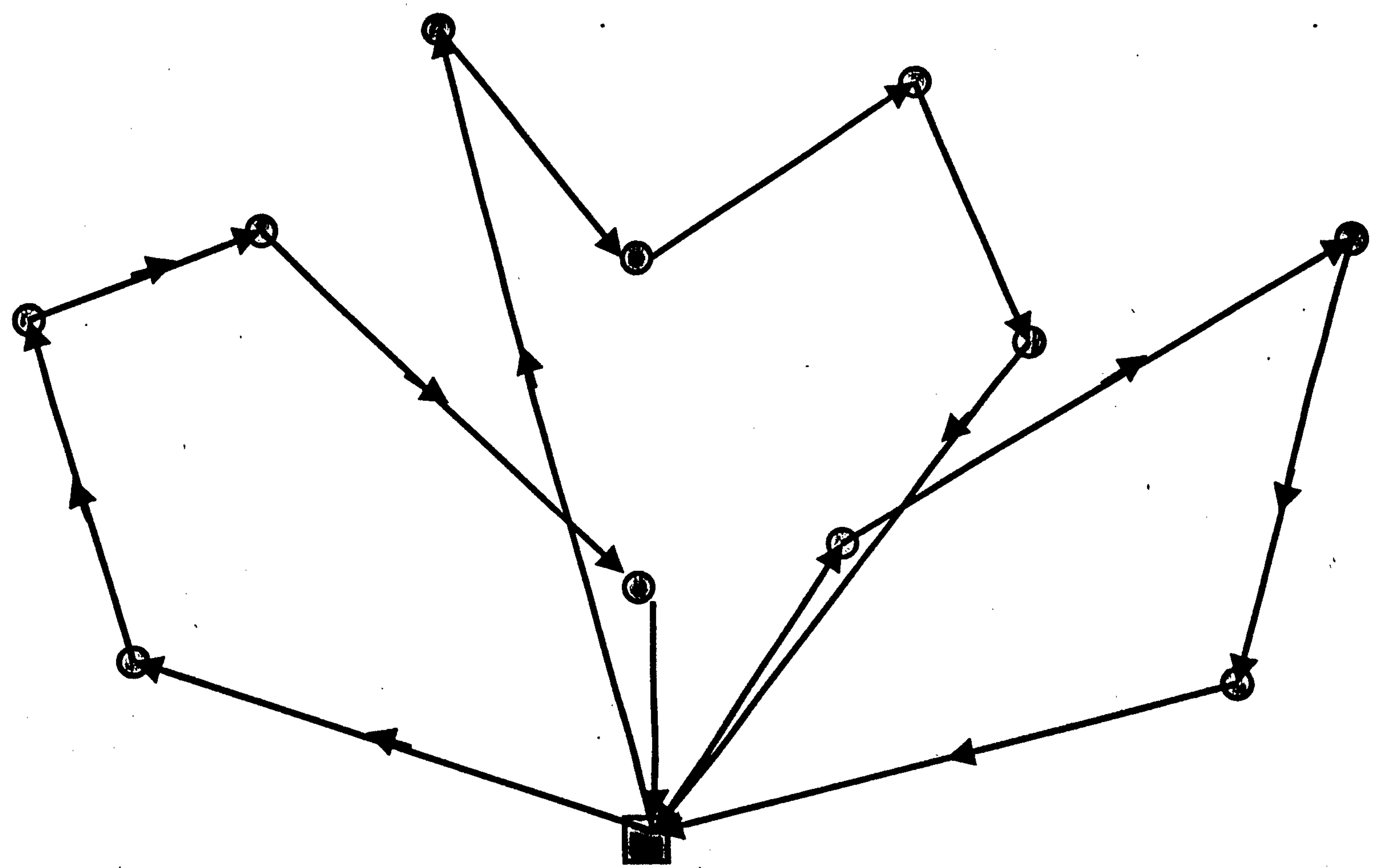

Fig.1. Problem of observation. **o**– expected location of probabilistic targets or observation regions; **n** – location of observation vehicle; --- possible paths of observation vehicles.

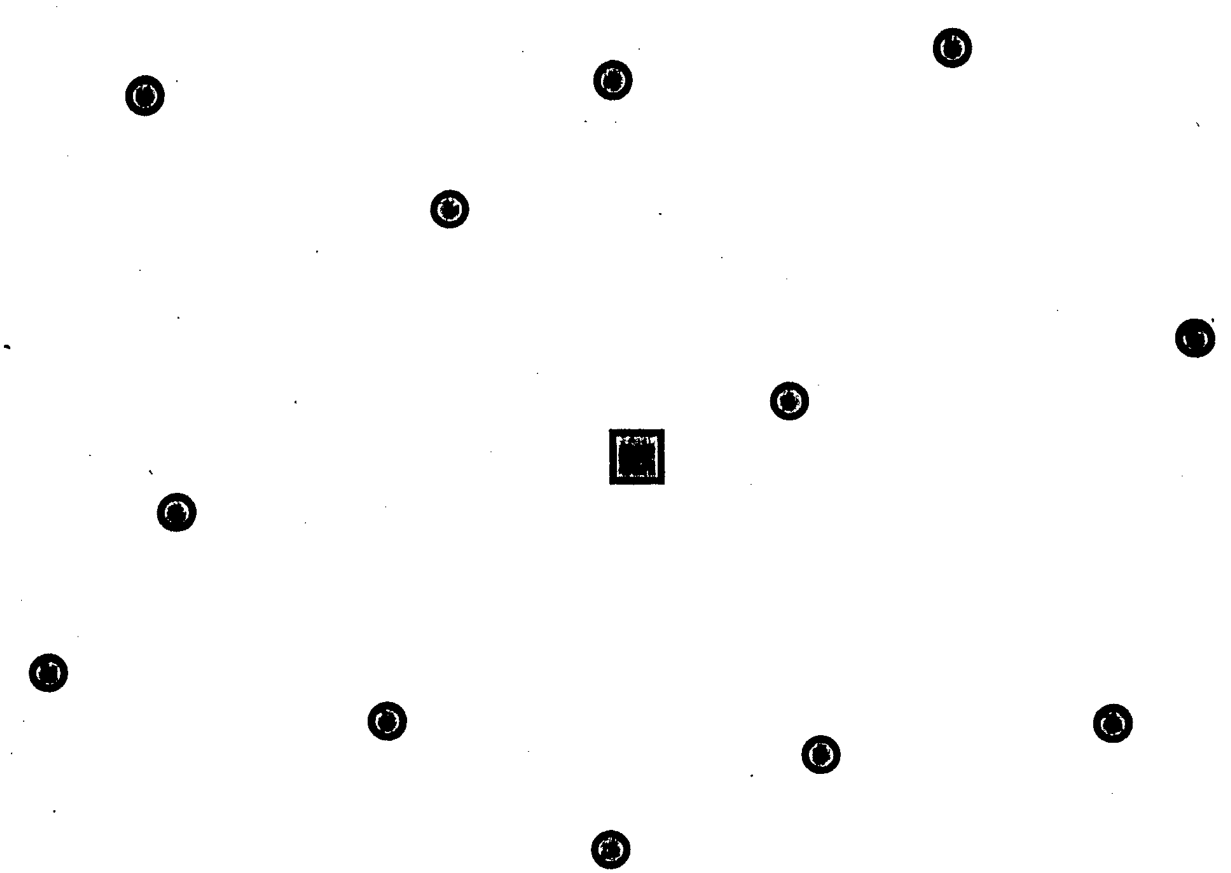

Fig.2. Optimal observer location of a single attack vehicles.

Fig_3_SS

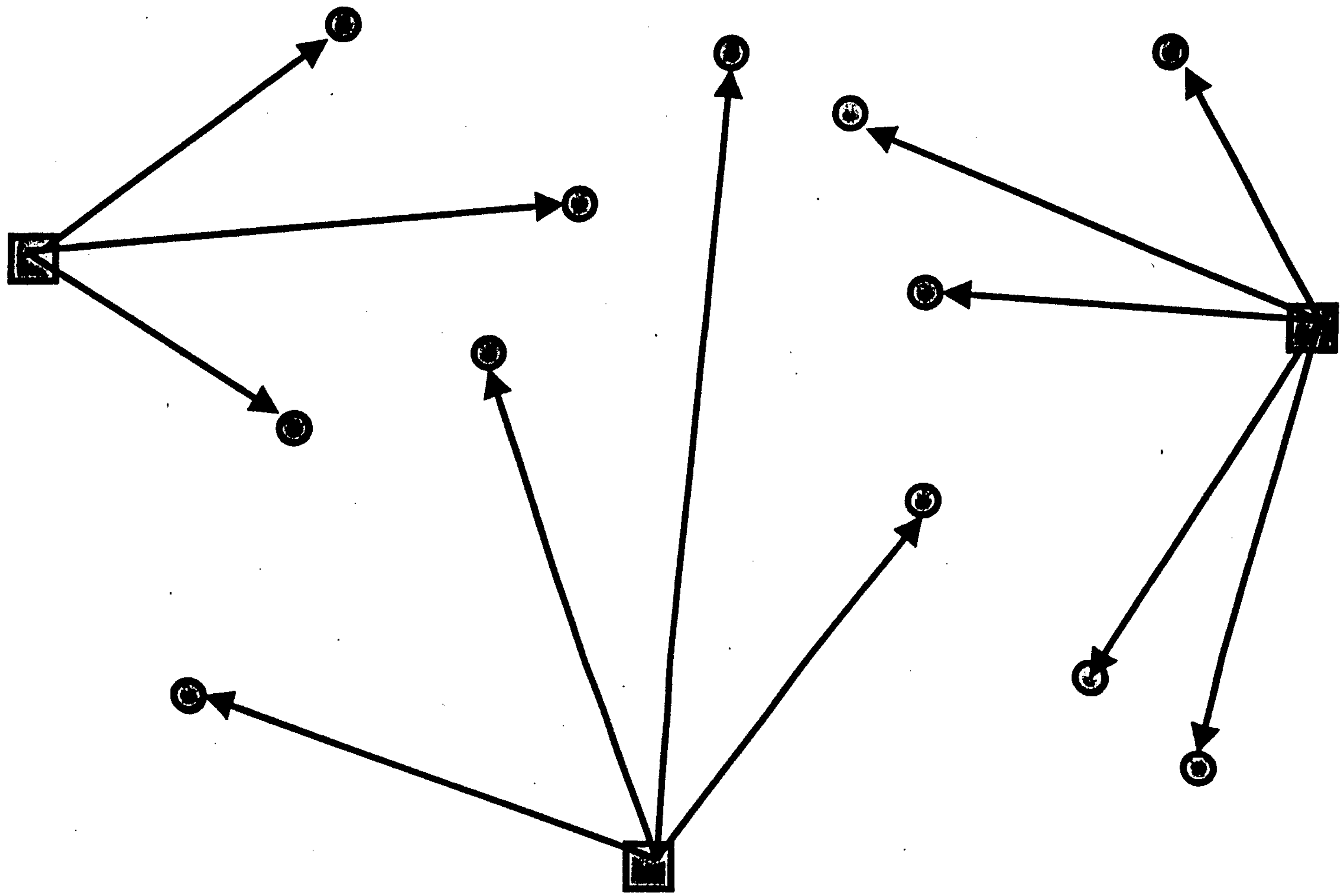

Fig.3. The problem of the quick annihilation of targets by attack vehicles located in different bases.

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